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Name	
Roll No	
Program	BCA
Course Code	DCA2101
Course Name	COMPUTER ORIENTED NUMERICAL METHODS
Semester	III

Question .1.a.) Show that

$$\textbf{(a)} \quad \boldsymbol{\delta\mu = \frac{1}{2}(\Delta + \nabla)}$$

Answer.:- The Laplacian operator in Cartesian coordinates is given by .:-

$$\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The gradient operator in Cartesian coordinates is .:-

$$\Delta = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

The Kronecker delta , δ, is defined as

$$\delta ij = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases}$$

Now , let's evaluate $\delta\mu$ in terms of $\Delta\ and\ \nabla$.:-

$$\delta\mu = \frac{1}{2}(\Delta + \nabla)$$

In Cartesian coordinates , Kronecker delta μ = μ is 1 , and for μ ≠ μ is 0 .

Let's consider the components of $\delta\mu$.:-

When $\mu = \mu$:

$\delta\mu\mu = 1$

When $\mu = \mu$:

$\delta\mu\mu = 0$

Therefore , the expression $\delta\mu\mu = \frac{1}{2}(\Delta + \nabla)$ doesn't hold true for arbitrary μ .

$$\textbf{(b)} \quad \boldsymbol{\Delta - \nabla = \Delta\nabla}$$

Answer.:- Given $\Delta - \nabla = \Delta\nabla$

Let's use the definition of the Laplacian and gradient operators:

The Laplacian operator in Cartesian coordinates is $\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

The gradient operator in Cartesian coordinates is $\Delta = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$.

Substitute these definitions into the equation:

$$\nabla^2 = \nabla = \nabla(\nabla)$$

Let's solve this step by step:

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$

$\nabla(\nabla) = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right) \quad \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)$

$\nabla(\nabla) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\hat{\imath}\right) + \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\hat{\jmath}\right) + \frac{\partial}{\partial x}\left(\frac{\partial}{\partial z}\hat{k}\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\hat{\imath}\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\hat{\jmath}\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}\hat{k}\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}\hat{\imath}\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}\hat{\jmath}\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\hat{k}\right)$

It appears that $\Delta - \nabla$ is not equal to $\Delta\,\nabla$ based on thew definitions of the Laplacian and gradient operators . The result dose not simplify to the from $\Delta - \nabla = \Delta\,\nabla$.

Therefore ,

As it stands , the equation $\boldsymbol{\Delta} - \boldsymbol{\nabla} = \boldsymbol{\Delta}\,\boldsymbol{\nabla}$ dose not hold true .

Question .2.) Solve the system of equation by Gauss Elimination's method

$$\mathbf{2x + y + 4z = 12}$$

$$\mathbf{4x + 11y - z = 33}$$

$$\mathbf{8x - 3y + 2z = 20.}$$

Answer.:- The given system of equations is:

1. $2x + y + 4z = 12$
2. $4x + 11y - z = 33$
3. $8x - 3y + 2z = 20$

Let's start by representing this system as an augmented matrix:

$$\left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 2 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right)$$

Now, let's perform row operations to bring this matrix to row-echelon form:

R2 = R2 - 2 * R1

R3 = R3 - 4 * R1

$$\left(\begin{array}{ccc|c} 2 & 1 & 12 & 12 \\ 0 & 9 & 9 & 33 \\ 0 & 0 & -21 & -21 \end{array}\right)$$

Now, let's solve for z from the last row:

-21 z = -21

z = 1

Substituting z=1 into the second row:

9 y - 9 = 9

9 y 18

Y = 2

Finally , substituting z = 1 y = 2 into the first row :

2 x + 2 + 4 = 12

2 x = 6

X = 3

Therefore ,

The solution to the system of equations is x = 3 , y = 2 , and z = 1 .

Question .3.) Find the equation of the best fitting straight line for the data:

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

Answer.:-

Given data:

X: 1, 3, 4, 6, 8, 9, 11, 14

Y: 1, 2, 4, 4, 5, 7, 8, 9

Calculations:

n = 8

$$\sum x = 1 + 3 + 4 + 6 + 8 + 9 + 11 + 14 = 56$$

$$\sum y = 1 + 2 + 4 + 4 + 5 + 7 + 8 + 9 = 40$$

$$\sum xy = (1*1) + (3*2) + (4*4) + (6*4) + (8*5) + (9*7) + (11*8) + (14*9) = 416$$

$$\sum x^2 = (1^2) + (3^2) + (4^2) + (6^2) + (8^2) + (9^2) + (11^2) + (14^2) = 494$$

Now, let's calculate the slope (m) using the formula:

$$m = \frac{(8 \times 416) - (56 \times 40)}{(8 \times 494) - (56)^2} = \frac{3328 - 2240}{3952 - 3136} = \frac{1088}{816} = \frac{34}{26} = \frac{17}{13}$$

Next, calculate the y-intercept (c):

$$c = \frac{40 - \frac{17}{13} \times 56}{8} \frac{40 - \frac{952}{13}}{8} = \frac{120}{13}$$

Therefore,

The equation of the best-fitting straight line for this data set is:

$$\boldsymbol{y = \frac{17}{13}x + \frac{120}{13}}$$

Set - II

Question .4.) Evaluate *f* (15), given the following table of values:

x	10	20	30	40	50
y = *f* (*x*)	46	66	81	93	101

Answer.:-

Given the points (x, y):

(10, 46) and (20, 66)

The formula for linear interpolation between two points is:

$$f(x) = f(x_0)\frac{(x - x_0).(f(x_1) - f(x_0))}{x_1 - x_0}$$

Where .:-

x_0 *and* $f(x_0)$ *are the values at the point before the target value* (*in this case*, $x = 10$ *and* $f(x) = 46$)

x_1 *and* $f(x_1)$ *are the values at the point before the target value* (*in this case*, $x = 20$ *and* $f(x) = 66$)

and $f(x) = 66)$,

and x *is the target value* (*in this case* , $x = 15$) .

let's plug in the values .: –

$$f(15) = 46 + \frac{(15 - 10).(66 - 46)}{20 - 10}$$

$$f(15) = 46 + \frac{5.20}{10}$$

$$f(15) = 46 + 10$$

$$f(15) = 56$$

So ,

Based on linear interpolation from the given table , $\boldsymbol{f(15) = 56}$

Question .5.) Use Taylor's series method to solve the initial value problem:

$$\frac{dy}{dx} = x^2 + y^2 \; for \; x = 0.25 \; and \; 0.5 \; given \; that \; y(0) = 1 .$$

Answer.:-

The differential equation given is:

$$\frac{dy}{dx} = x^2 + y^2$$

Given the initial condition $y(0) = 1$, we'll start by finding the Taylor series expansion *for y (x)* around $x = 0$:-

Let $y(x) = y_0 + y_1 x + y_2 x^2 + y_3 x^3 + \ldots\ldots\ldots$

$$\frac{dy}{dx} = y1 + 2y_2 x + 3y_3 x^2 \ldots\ldots\ldots$$

Using the given differential equation, we get:

$$y_1 + 2y_2 x + 3y_3 x^2 + \ldots\ldots\ldots x^2 + (y_0 + y_1 x + y_2 x^2 + y_3 x^3 + \ldots\ldots)^2$$

To solve for the coefficients, equate the corresponding powers of x on both sides of the equation.

At x = 0 :

$y_1 = 0^2 + y_0^2 (using \; the \; intial \; condition \; y(0) = 1$

So , $y_1 = 1$.

At x = 0 :

$y_1 = 2y_2(0) = 0^2$

So , $y_2 = 0$.

We've found the first two coefficients. Now, we can express y(x) using the Taylor series expansion:

Substituting the values we found:

y (x) = 1 + x

Now, we'll use this approximation to find the values of Y AT X = 0.25 and x = 0.5 :

Y (0 . 25) = 1 + 0.25 = 1.25

Y (0.5) = 1 + 0.5 = 1.5

So, using Taylor's series method, the approximated values of y at x = 0.25 and x = 0.5 are 1.25 and 1.5 respectively , based on the first – order Taylor series expansion around x = 0 .

Question .6.) Apply Runge-Kutta fourth order method to find an approximate value of y when x = 0.1 given that $\frac{dy}{dx} = x^2 - y, \;\; y(0) = 1.$

Answer .:-

Given the initial condition $\frac{dy}{dx} = x^2 - y, \;\; y(0) = 1$, we need to solve for y at x = 0.1

The fourth-order Runge-Kutta method involves the following steps:

1. Define the step size , h .
2. Using following formulas to iterate and approximate y :

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$
$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$
$$k_4 = hf(x_n + h, y_n + k3)$$
$$y_n + 1 = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + 2k_4)$$

$where .: -$

- h is the step size .
- x_n and y_n are the known x and y values at the current step .
- F(x,y) is the differential equation : x^2 – y .

Let's implement this algorithm to find the approximate value of y when x = 0.1 we'll start with h = 0.1 for this example .

Let me recompute the solution using the fourth-order Runge-Kutta method.

Given the differential equation $\frac{dy}{dx} = x^2 - y$ and the initial condition y(0) = 1 , we want to find the approximate value of y at x = 0.1 .

1. Define the function f(x,y) based on the given differential equation .:-
2. Using the following formulas to iterate and approximate y :

$$k_1 = h \, . \, f(x_n, y_n)$$
$$k_2 = h \, . \, f(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$
$$k_3 = h \, . \, f(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$
$$k_4 = h \, . \, f(x_n + h, y_n + k3)$$
$$y_n + 1 = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + 2k_4)$$

Let's compute the approximate value of y at x 0.1 using the given method and initial conditions .
Let's use the fourth-order Runge-Kutta method to approximate the value of y where x = 0.1 for the given differential equation :-

$\frac{dy}{dx} = x^2 - y$

with the initial condition y (0) = 1.
The fourth-order Runge-Kutta method involves iterative steps. We'll use a step size h and perform calculations to approximate y at x = 0.1. The

formulas for the method are:

1. $k_1 = h \, . f(x_n, y_n)$
2. $k_2 = h \, . f(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
3. $k_3 = h \, . f(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$
4. $k_4 = h \, . f(x_n + h, y_n + k3)$

   Where f(x, y) = x^2 y , and then :

$$\boldsymbol{y_n + 1 = \ y_n \frac{1}{6}(k_1 + 2k_2 + 2k_3 + 2k_4)}$$